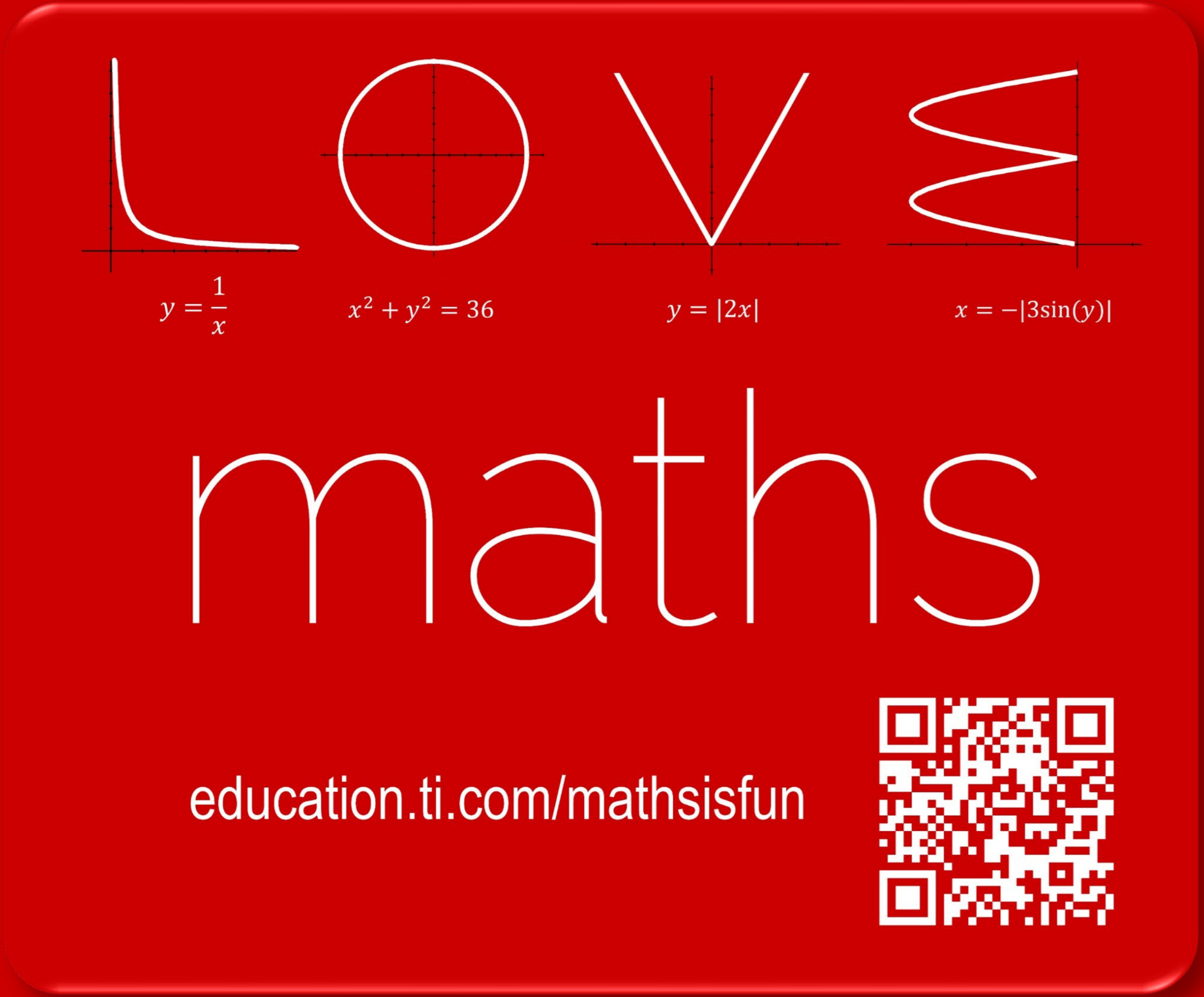
REAL OR ARTIFICIAL INTELLIGENCE

Mind blowing puzzles

These puzzles and problems presented in this workshop represent a subset of the free puzzles via the mouse pad given away at the 2024 MAV Conference.



Random chords

Wh chords

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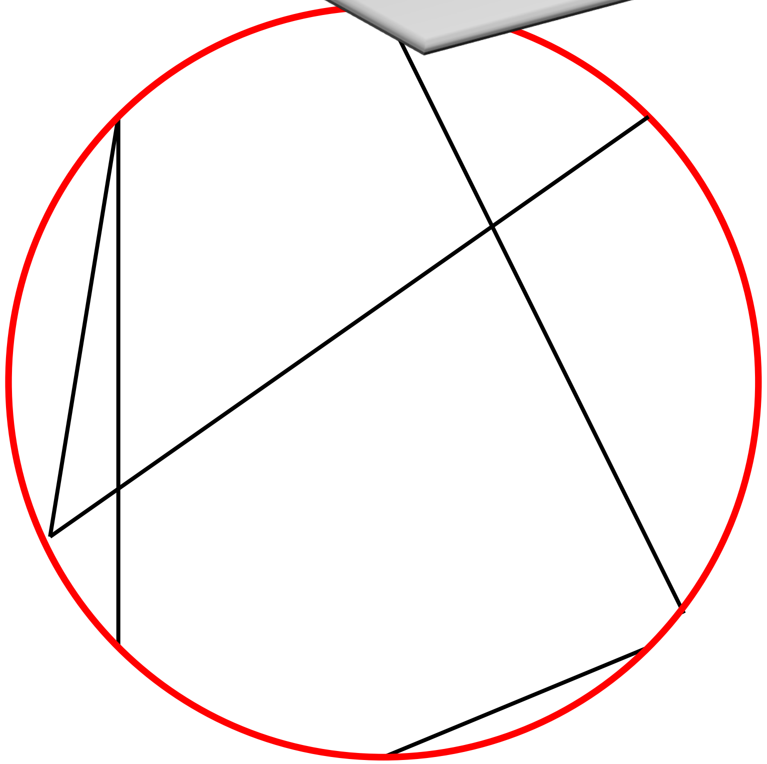
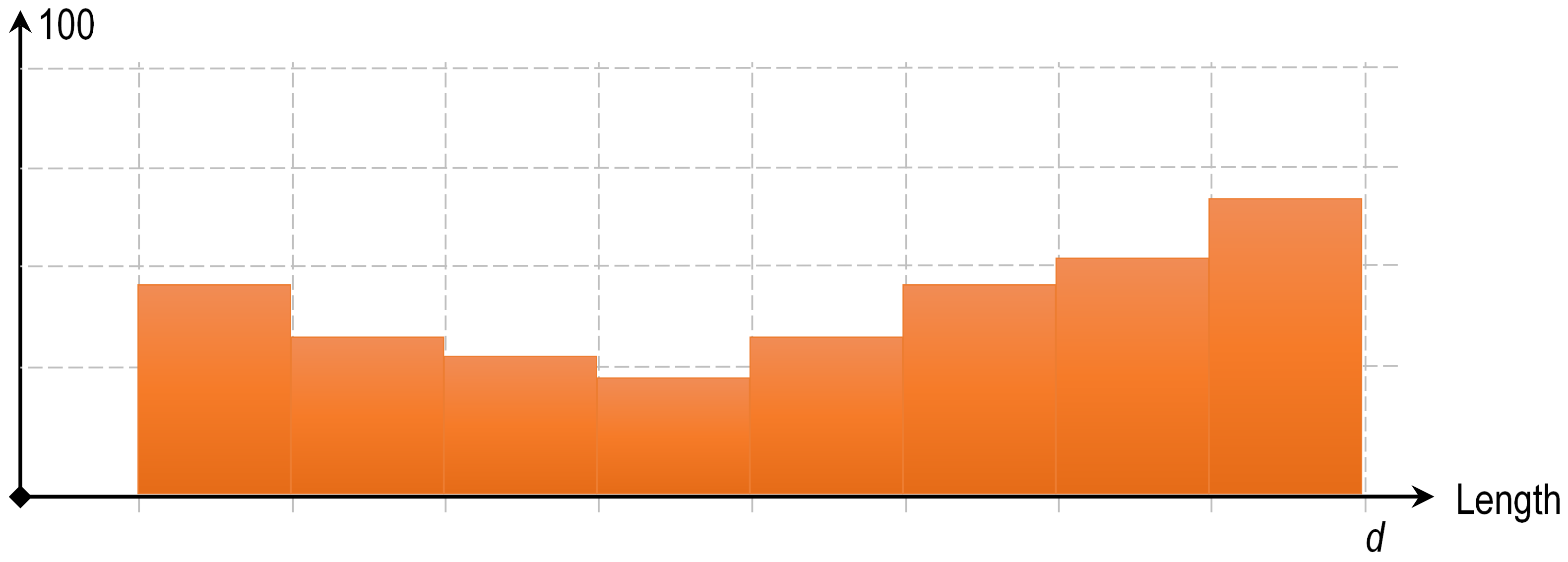
Draw four random chords in the circle.
Measure the length of each (mm).



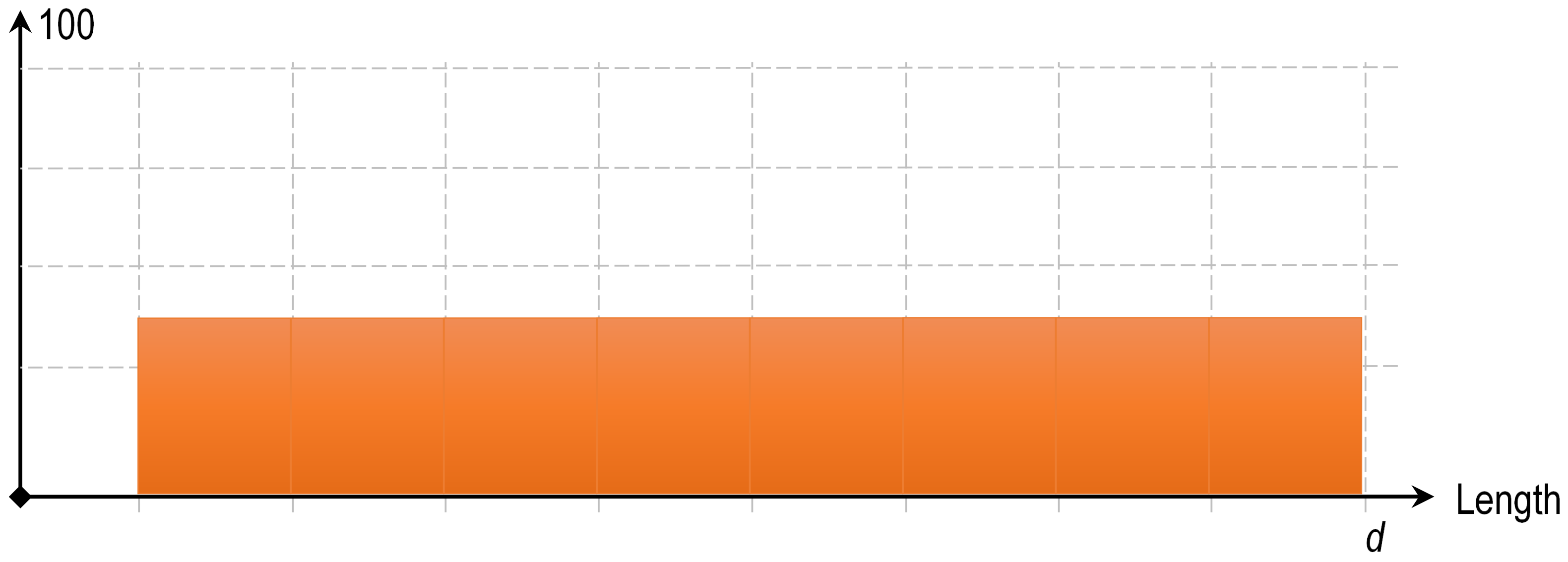
Random chords

What would the distribution of chord lengths look like?



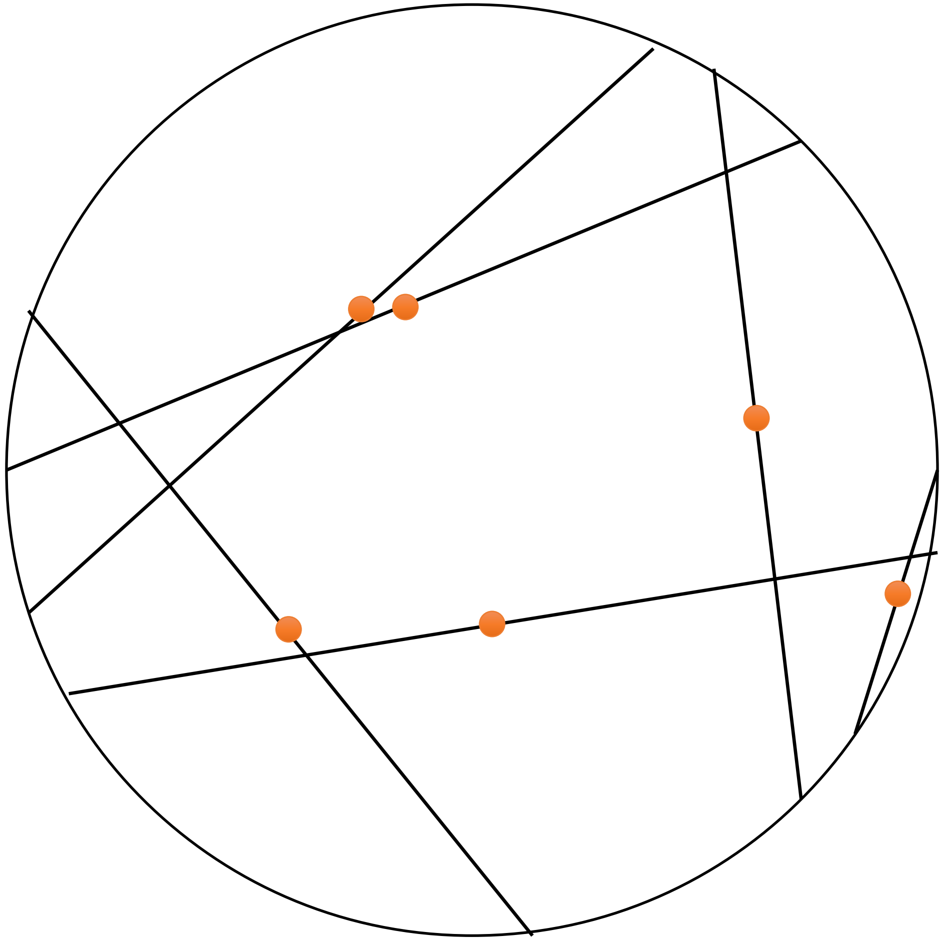
Random chords

What would the distribution of chord lengths look like?



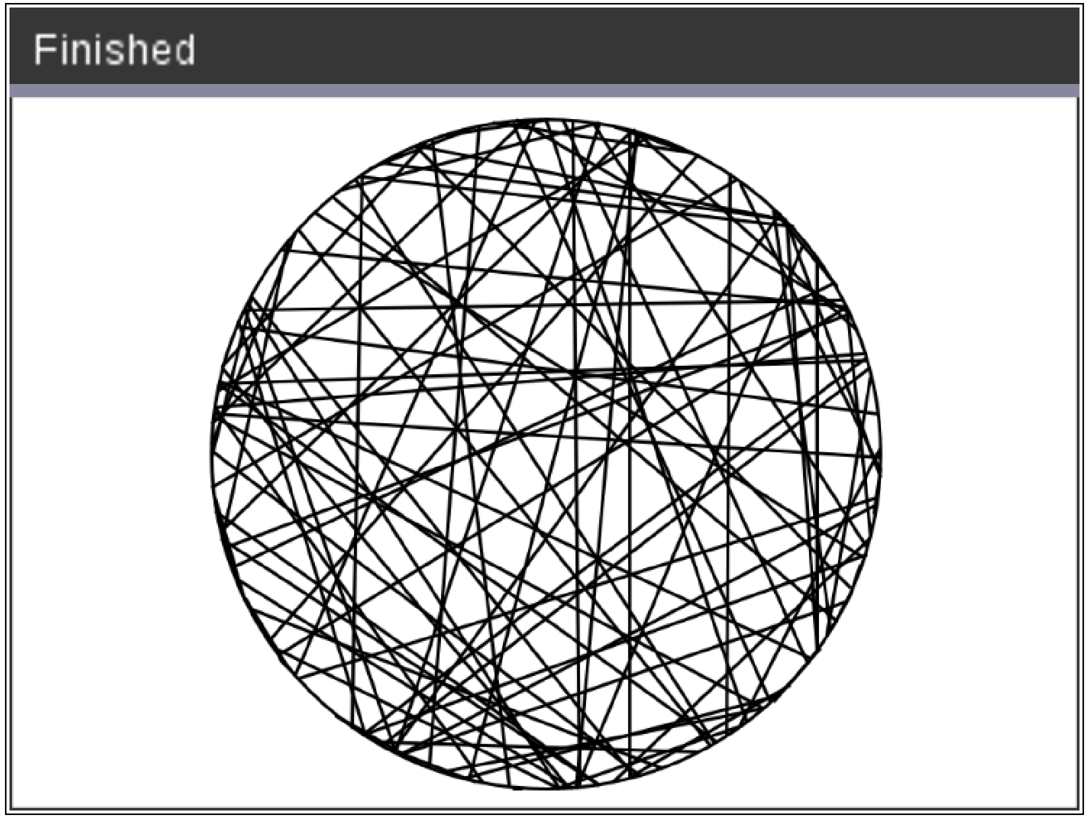
Random chords

What about the distribution of midpoints for each chord?



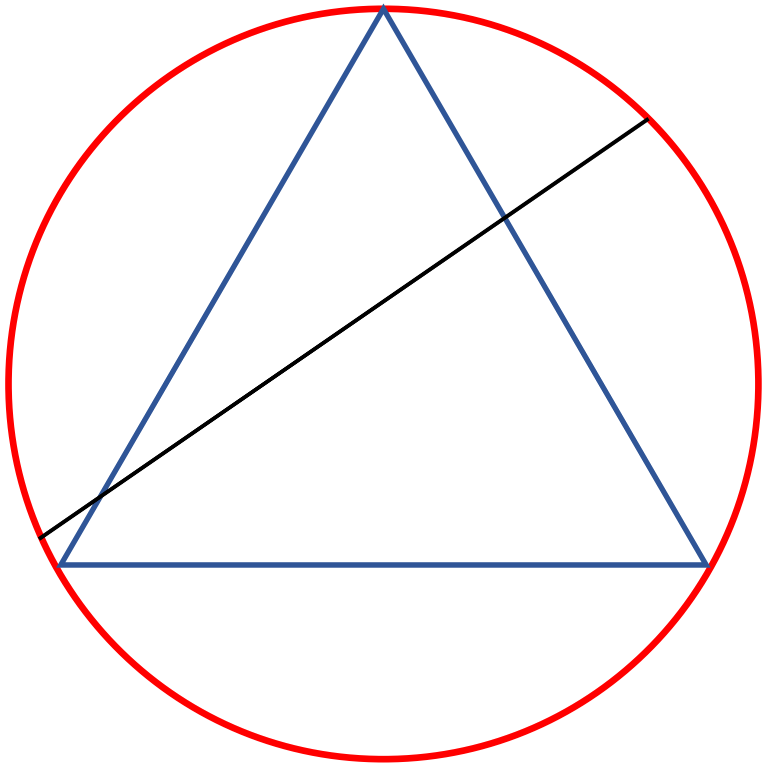
Random chords: Part 1

Let’s generate 100+ chords …



Random chords

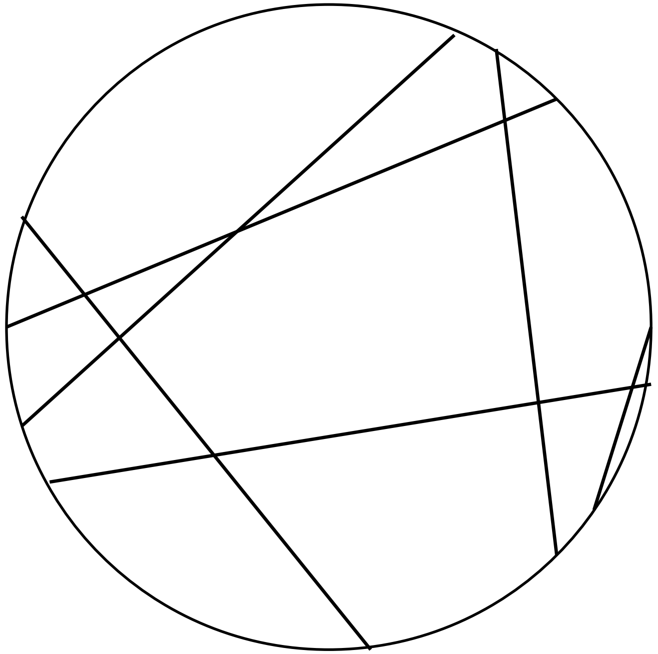
What proportion would be longer than the side lengths of an inscribed (equilateral) triangle?



Random chords: Part 1

How confident are you with your final result?

- » Visual distribution of the chords
- » Proportion generated by simulation
- » Theoretical result (Geometric Proof)
- » Distribution of chord lengths
- » Distribution of chord midpoints

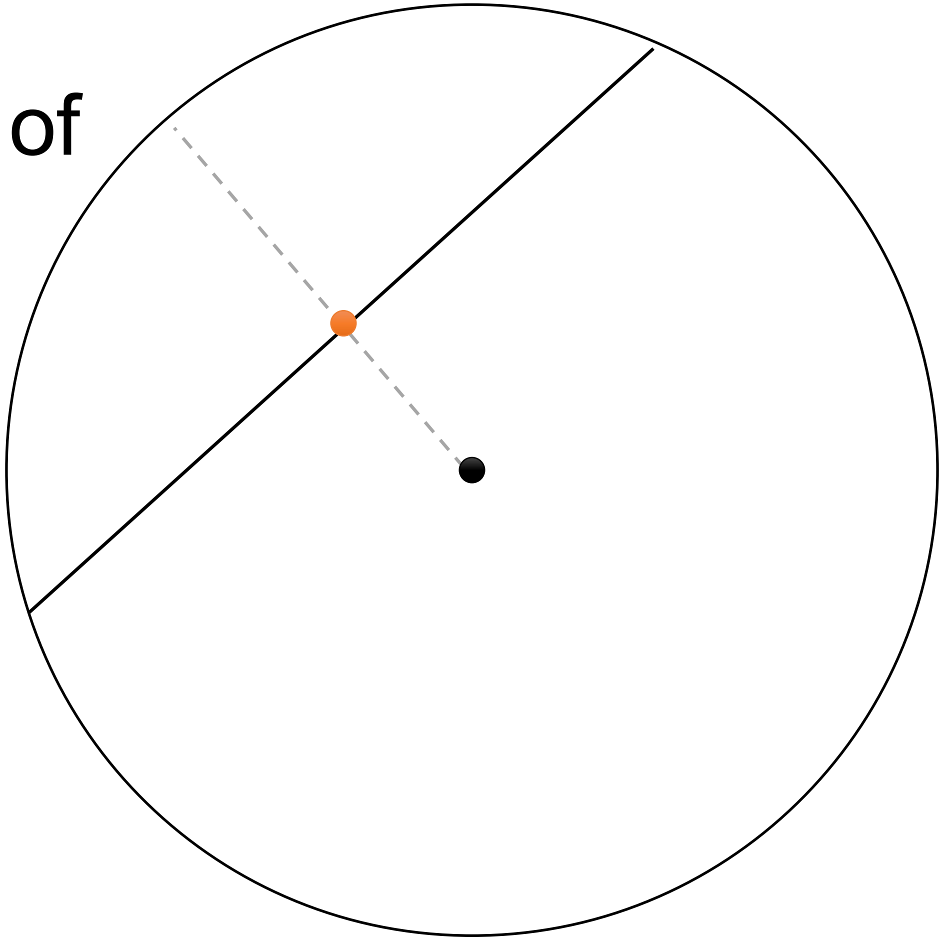


Random chords: Part 2

» A point is generated at random.

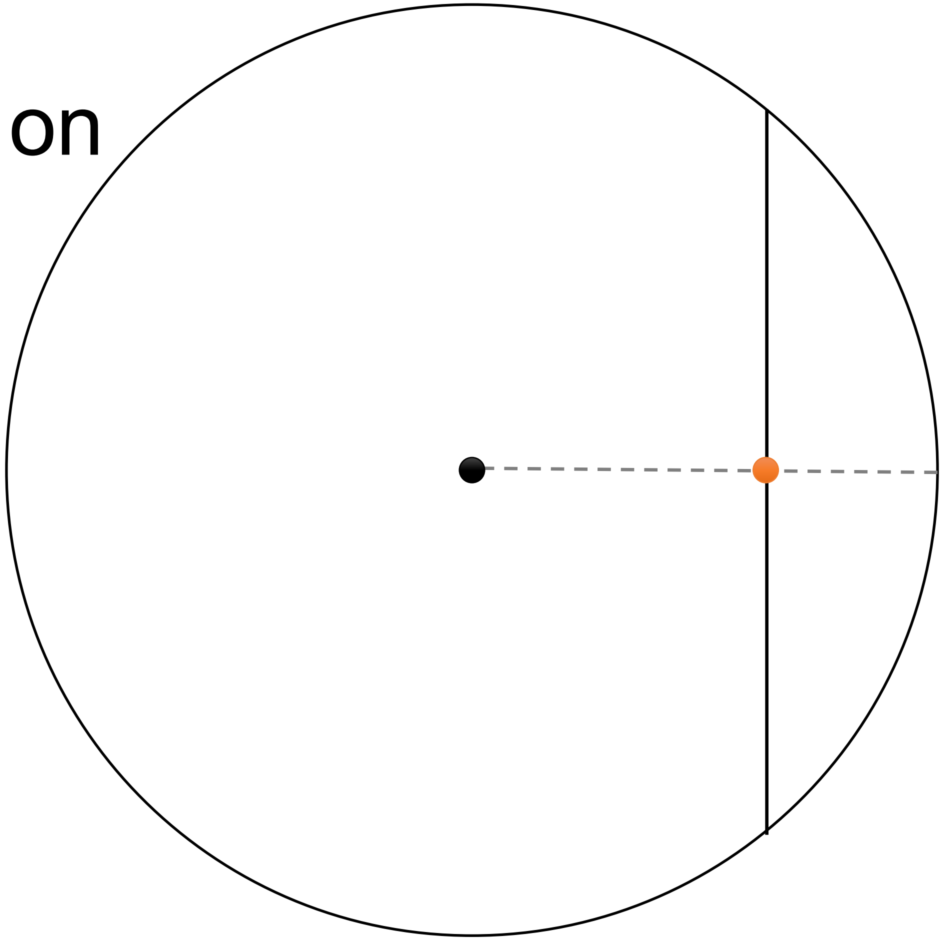
» It is the midpoint of a random chord.

» Generate two chords in each circle based on this technique.

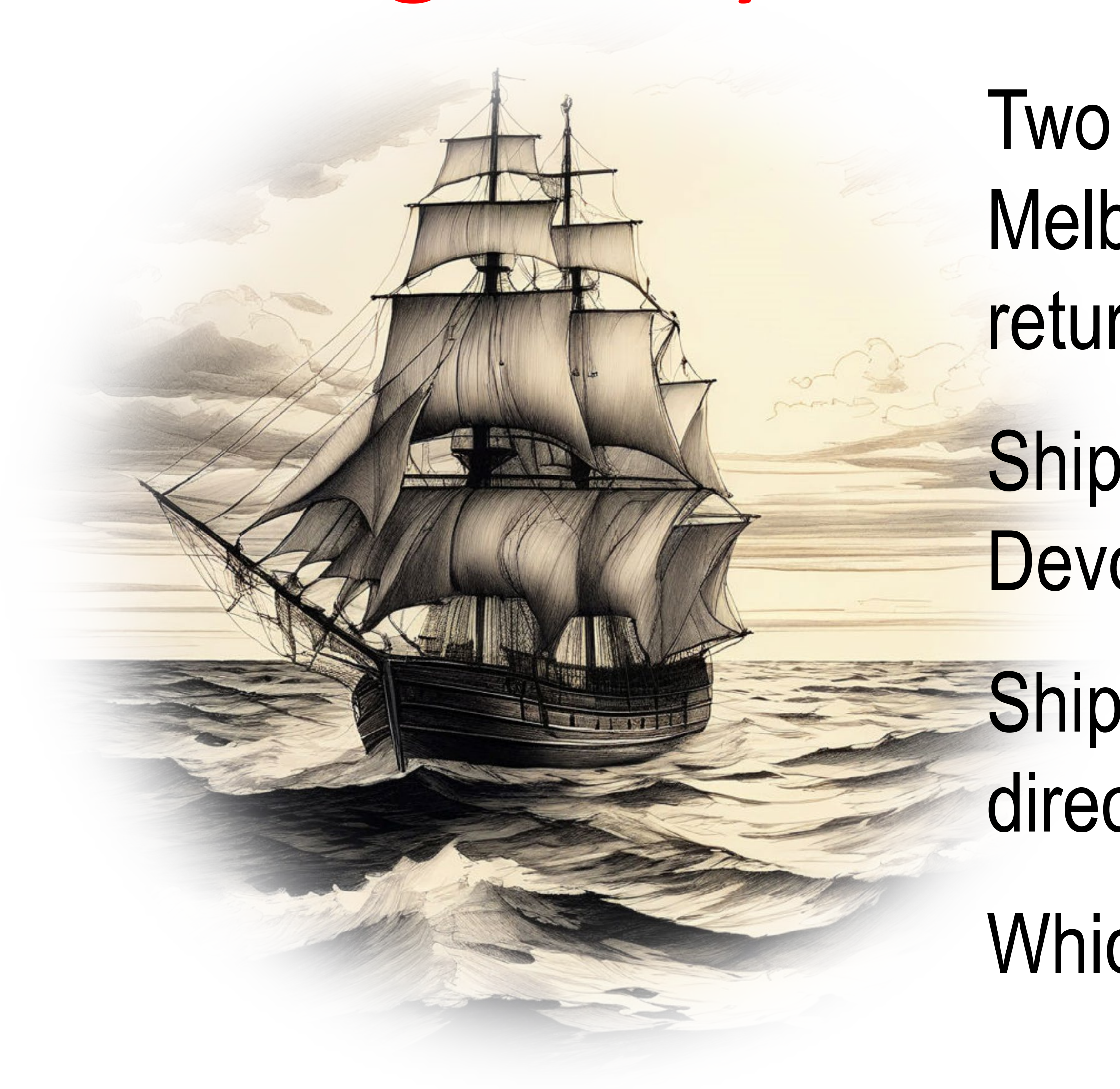


Random chords: Part 3

- A line is generated along the radius.
- A point is placed on the line.
- The chord is constructed.



Sailing Away



Two sailing ships leave simultaneously from Melbourne, bound for Devonport before returning immediately to Melbourne.

Ship A sails at speed of 30 knots on the way to Devonport and 40 knots on the return journey.

Ship B sails at a speed of 35 knots in both directions.

Which ship will return to Melbourne first?

Sailing Away

Let x = distance from Melbourne to Devonport.
Calculate the total travel time:

Ship A: $\dfrac{x}{30}+\dfrac{x}{40}=\dfrac{7x}{120}$

Ship B: $\dfrac{x}{35}+\dfrac{x}{35}=\dfrac{2x}{35}$

Since: $\dfrac{2x}{35}<\dfrac{7x}{120}$ ship B wins!

Saving Money

Andrew and Mary are both trying to save $120.

Andrew saves at the rate of $10 per week until he reaches $60; after which he saves at a rate of $20 per week.

Mary saves at the rate of $15 per week until she reaches her goal.

Do Andrew and Mary reach their goal at the same time, or does one of them reach their goal first?

Saving Money

Andrew and Mary are both trying to save $120.

Andrew saves at the rate of $20 per week until he reaches $60; after which he saves at a rate of $10 per week.

Mary saves at the rate of $15 per week until she reaches her goal.

Do Andrew and Mary reach their goal at the same time, or does one of them reach their goal first?

Makings of a half

Using all the digits 1, 2, 3 … 9 (no repeats),

write a fraction that is equal to ½.

1 2 3 4 5 6 7 8 9

Makings of a half

Using all the digits 1, 2, 3 … 9 (no repeats),

write a fraction that is equal to ½.

$$\frac{6\ 3\ 5\ 7}{1\ 2\ 8\ 9\ 4}$$

?

Problem Selection

Selection of Problems

Number problems where manipulatives are particularly beneficial.

1 2 3 4 5 6 7 8 9

Problems for Senior Maths

Beautiful problems involving functions:
Linear, Quadratic, Cubic and Hyperbola.

Nine Times Reverse

A four digit number is multiplied by 9, the result (answer) contains the same four digits but in the reverse order.

What is the original number?

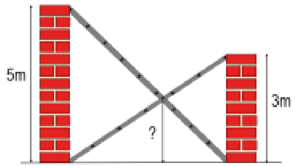
Fraction Challenge

$$\frac{44}{37} = 1 + \cfrac{1}{a + \cfrac{1}{b + \cfrac{1}{c}}}$$

Find the values for a, b and c given each is a positive integer.

Ladders

Two ladders lean against two walls. One wall is 5m tall, the other 3m.
How high off the ground is it where the ladders cross?



Hooray for 2024 array

If the array shown below is continued, what numbers will appear directly above and below 2024?

1
2 3 4
5 6 7 8 9
10 11 12 13 14 15 16

What's my Number

I have a three-digit number.

- If you add 1 to the number, the result is divisible by 3.
- If you add 2 to the number, the result is divisible by 5.
- If you add 3 to the number, the result is divisible by 7.

What is my number?

As easy as One – Two - Three

Nine tiles, numbered 1, 2, … 9 can be used to form three numbers, each number consisting of three digits.

- The second number is twice the first
- The third number is three times the first.

What are the three numbers?

How many solutions can you find?

1 2 3 4 5 6 7 8 9

Down the Mountain

Down the Mountain is an 11 player game.

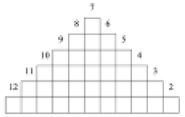
Players take it in turn to choose a number which is not already taken.

Once each person has a number, the croupier rolls two dice and adds the uppermost faces together.

A cross is then put under that number.

If a column is full of crosses that player wins, otherwise the dice are rolled again.

If you're the first player to choose a number, what number should you choose?



Swimming for Gold

In a 200m swimming race, the swimmer in lane 5 wins, beating the swimmer in lane 6 by 20m.

The swimmer in lane 6 beats the swimmer across in lane 4 by 15m.

Assuming all the swimmers are swimming at a constant rate, by how many metres did the winner (lane 5) beat the swimmer in lane 4?

Selection of Problems

Number problems where manipulatives are particularly beneficial.

1 2 3 4 5 6 7 8 9

Sum Tiles – Part 1

Ten tiles contain numbers:

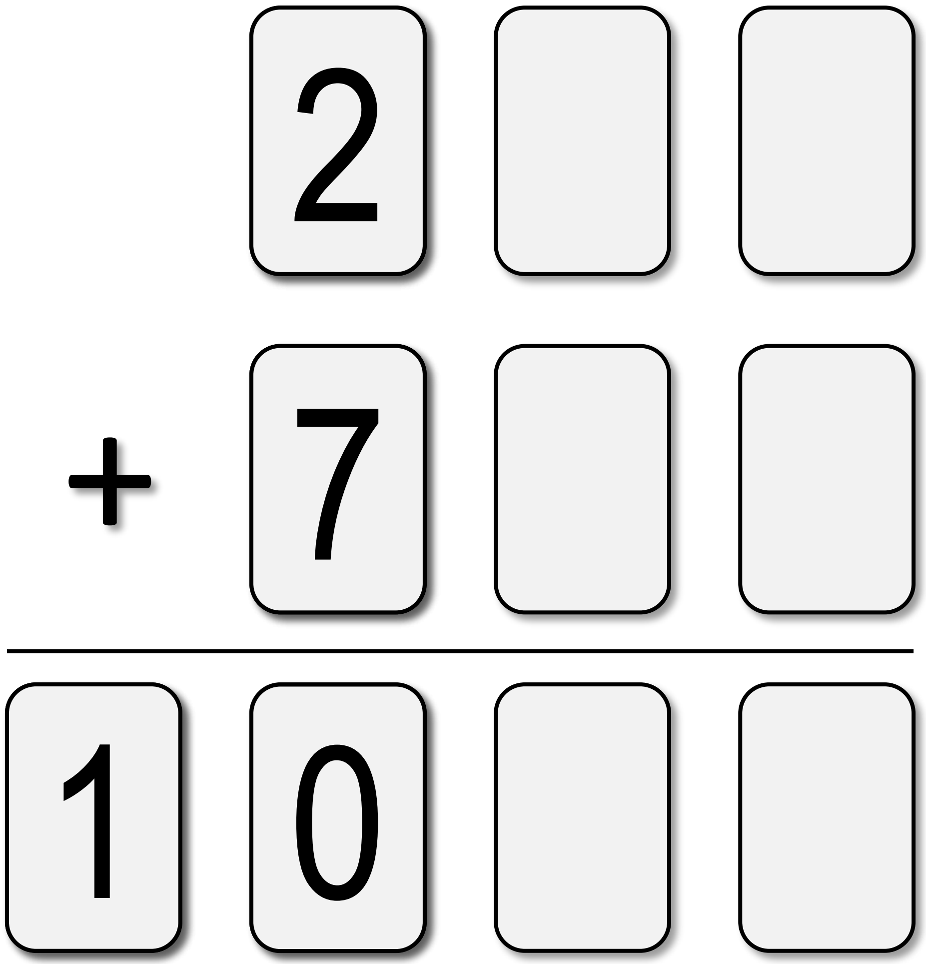
0, 1, 2, 3, 4, 5, 6, 7, 8 & 9.

The tiles can be arranged in such a way that the sum is true.

Can you find a solution?

How many solutions can you find?

What do you notice about the four digit sum?



Sum Tiles – Part 1 - Solution

Solution

Sum Tiles – Part 1 - Solution

Ten tiles contain numbers: 0, 1, 2, 3, 4, 5, 6, 7, 8 & 9.

There are 96 possible sums; many of them are re-arrangements. (Samples below)

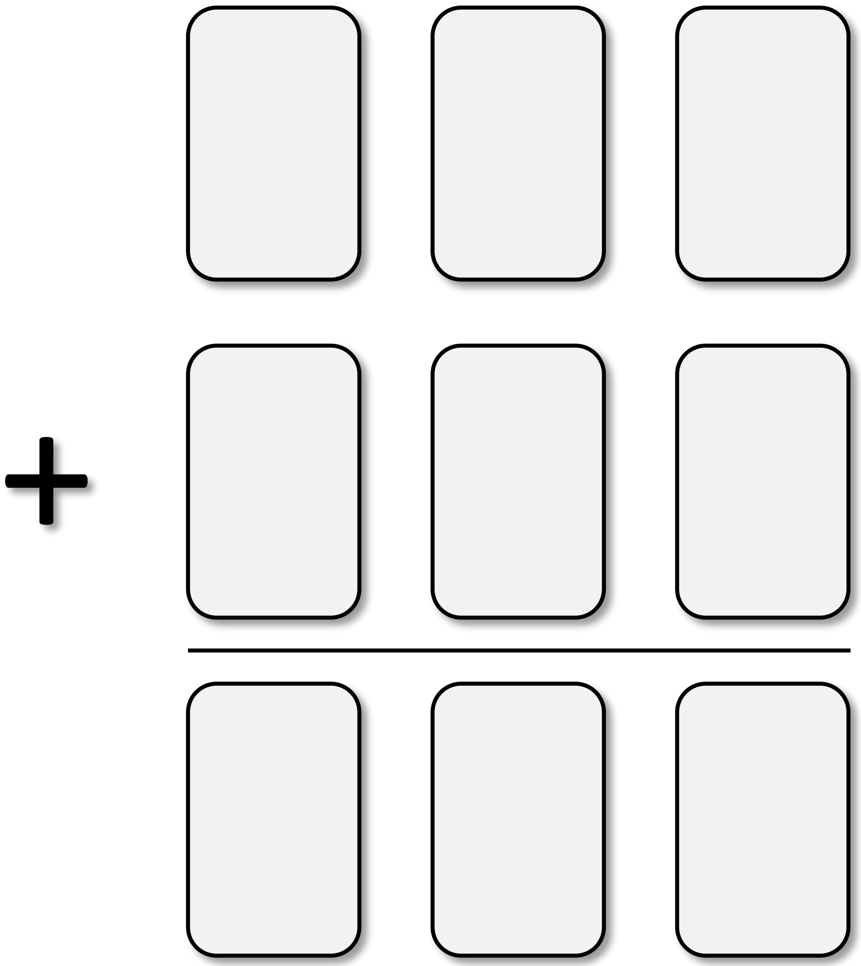
324	342	437	246	746
+ 765	+ 756	+ 652	+ 789	+ 289
1089	1098	1089	1035	1035

The first and second solution are simple re-arrangements of the tens and units columns. The third solution is slightly different as the 7 and 2 have swapped lines giving light to more solutions by rearranging..

Sum Tiles

Nine tiles contain numbers:
1, 2, 3, 4, 5, 6, 7, 8 & 9.
The tiles can be arranged such that the sum is true.

- Can you find a solution?
- How many solutions can you find?
- Emily notices that the final sum always seems to be divisible by nine. Comment on Emily's observation.



Sum Tiles - Solutions

There are 336 solutions (too many to show here). Sample solutions shown illustrate how digits can be rearranged to form similar solutions.

$$\begin{array}{r} 1\ 2\ 7 \\ +\ 3\ 6\ 8 \\ \hline 4\ 9\ 5 \end{array} \qquad \begin{array}{r} 1\ 2\ 8 \\ +\ 3\ 6\ 7 \\ \hline 4\ 9\ 5 \end{array} \qquad \begin{array}{r} 1\ 5\ 2 \\ +\ 4\ 8\ 7 \\ \hline 6\ 3\ 9 \end{array} \qquad \begin{array}{r} 1\ 5\ 2 \\ +\ 7\ 8\ 4 \\ \hline 9\ 3\ 6 \end{array}$$

It is not expected that a single student will find all the solutions, this could be a class project with student names assigned to each solution.

Discussion

How are manipulatives beneficial in these two problems?

1	2	3	4	5	6	7	8	9

Sum house numbers

Just for fun, Jess added up the house numbers in her street to the left or her house and got exactly the same result when she added up all the house numbers to the right.

Given there are more than 10 houses in the street:

- What is Jess's house number?
- How many houses does she have in her street?

Sum house numbers

Let n = Jess's house number and x the number of houses in the street.

Problem to solve: $\sum_{a=1}^{n-1} a = \sum_{a=n+1}^{x} a$ OR $\sum_{a=1}^{n} a = \sum_{a=n}^{x} a$

$$\frac{n(n-1)}{2} = \frac{x(x+1)-n(n+1)}{2}$$

Diophantine equation. This can be solved by substitution or by generating a table of values.

Solution 1: x = 8 , Jess's House = 6 & Solution 2: x = 49 , Jess's house = 35

Extension: What if she added up the houses on her side of the street only?

Problems for Senior Maths

Beautiful problems involving functions:
Linear, Quadratic, Cubic and Hyperbola.

Cool Locus

DOWNLOAD

Find the equation to the locus formed by the point of intersection of the altitudes of a triangle with vertices on the function:

$$y = \frac{1}{x}$$

Underline key terms

Draw a diagram

Cool Locus

Powerful Visual: Draw a dynamic diagram to see the locus.

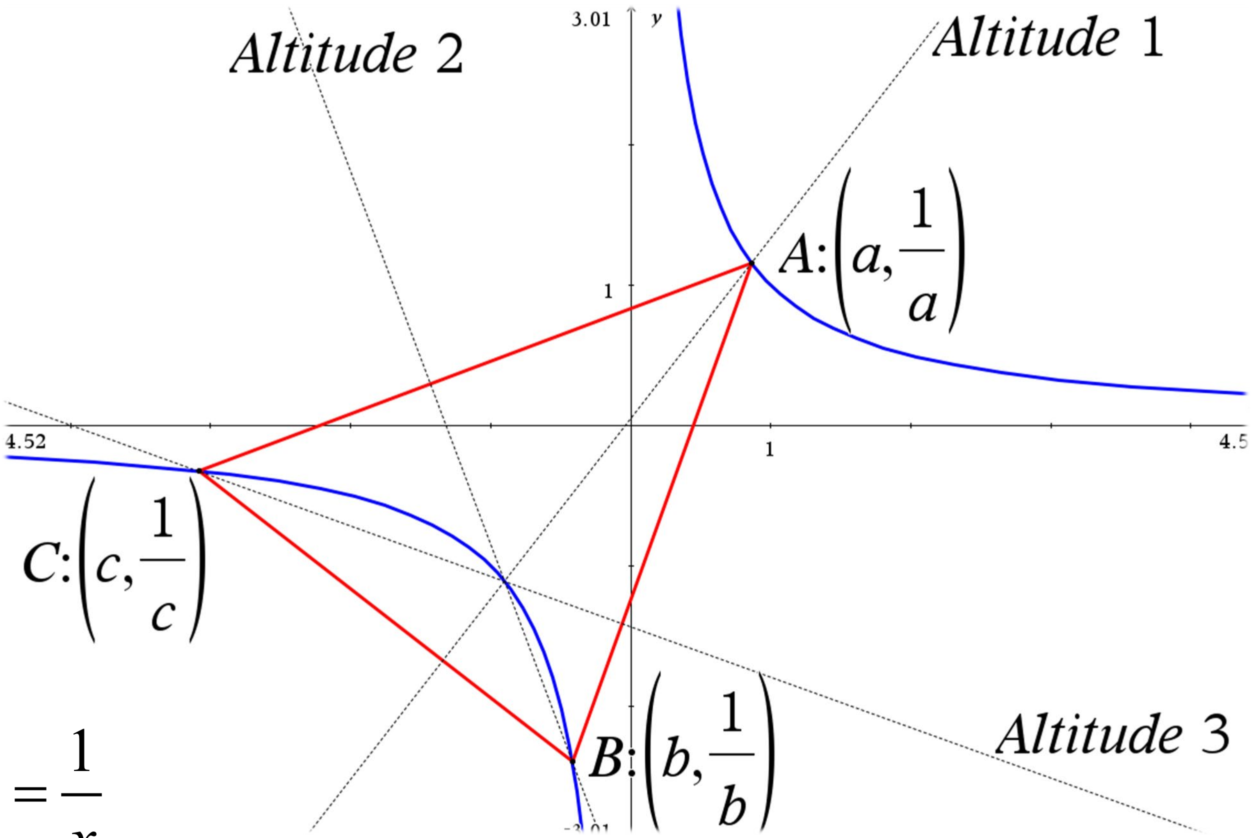
$$M_{\overline{BC}} = \frac{\frac{1}{b} - \frac{1}{c}}{b - c} \qquad M_{Altitude1} = bc$$

Altitude 1: $y = bc(x - a) + \frac{1}{a}$

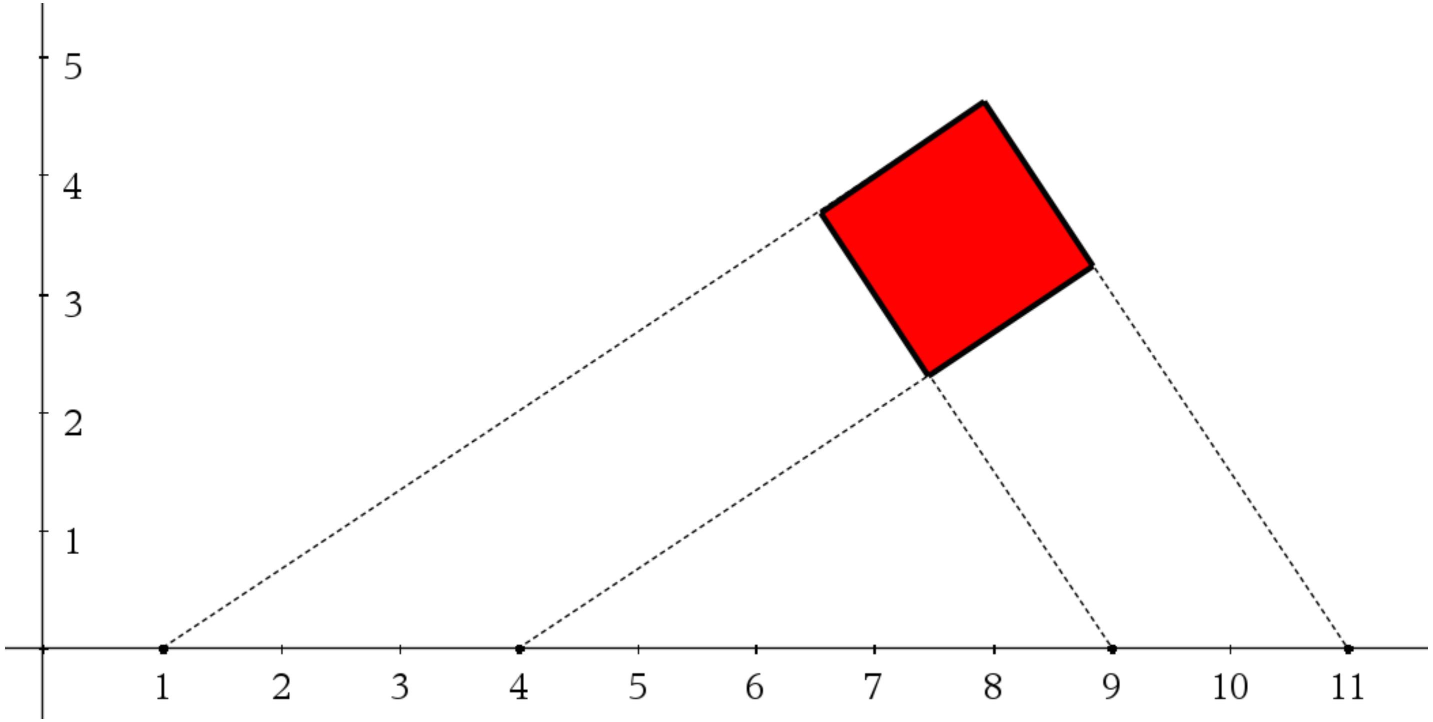
Altitude 2: $y = ac(x - b) + \frac{1}{b}$

Altitude 3: $y = ab(x - c) + \frac{1}{c}$

Simultaneous: $x = \frac{-1}{abc} \qquad y = -abc \qquad \therefore y = \frac{1}{x}$



Squared Up



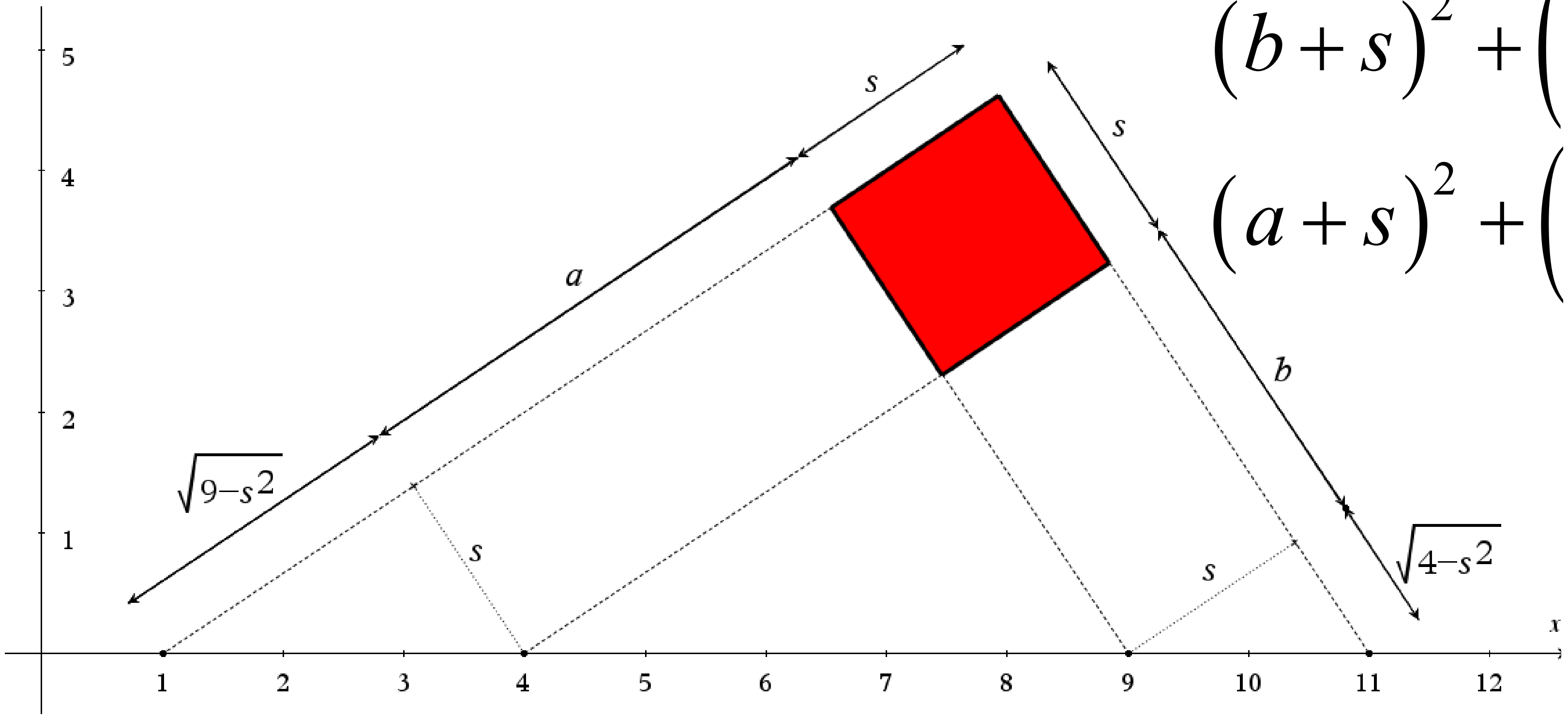
Determine the area of the shaded square.

Squared Up

Solution

There are numerous ways to solve this problem.
The following approach uses Pythagoras's theorem and simultaneous equations.

Squared Up



$$(b+s)^2+\left(a+\sqrt{9-s^2}\right)^2=64 \quad \textbf{Eqn1}$$

$$(a+s)^2+\left(b+\sqrt{4-s^2}\right)^2=49 \quad \textbf{Eqn2}$$

$$a^2+b^2=25 \quad \textbf{Eqn3}$$

Area of Square = $\dfrac{36}{13}$

There are numerous ways to solve this problem.
This approach uses Pythagoras's theorem and simultaneous equations.

Cubic Tangent

A cubic function $f(x)$ has three distinct x axis intercepts:

$$(a, 0), \ (b, 0), \ \& \ (c, 0).$$

A tangent to the function is drawn at the point: $x = \frac{a+b}{2}$

Determine where the tangent line crosses the x axis.

Cubic Tangent

SOLUTION

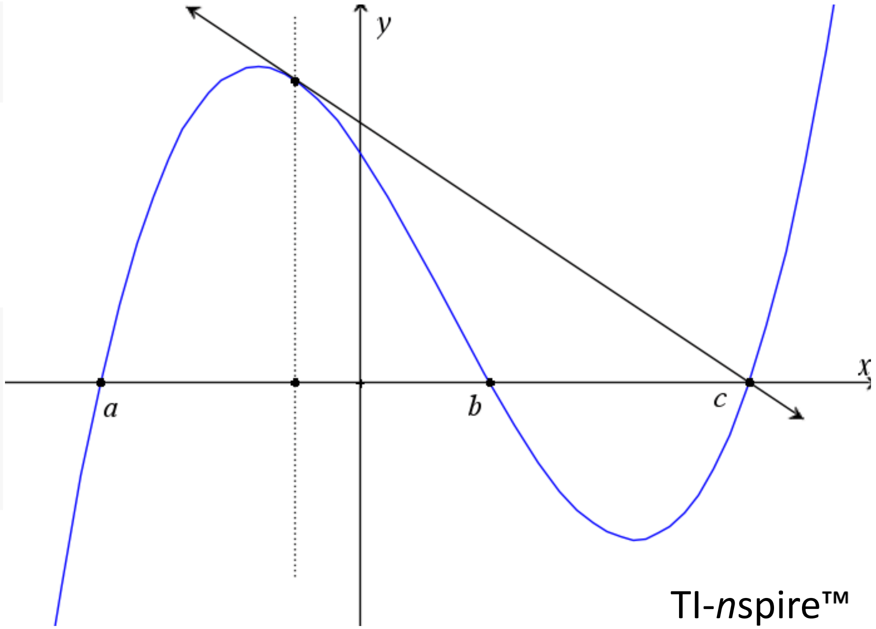
Cubic Tangent

Define $f(x)=(x-a)\cdot(x-b)\cdot(x-c)$ *Done*

$\frac{d}{dx}(f(x))|x=\frac{a+b}{2}$ $\frac{-a^2}{4}+\frac{a\cdot b}{2}-\frac{b^2}{4}$

Define $t(x)=\left(\frac{-a^2}{4}+\frac{a\cdot b}{2}-\frac{b^2}{4}\right)\cdot\left(x-\frac{a+b}{2}\right)+f\left(\frac{a+b}{2}\right)$ *Done*

$\text{solve}(t(x)=0,x)$ $x=c \text{ or } a^2-2\cdot a\cdot b+b^2=0$



TI-*n*spire™

The tangent passes through the third axis intercept (c,0). The question stated that "a, b and c are distinct" so the solution ($a = b$) is not applicable, however if $b = c$, the tangent would be the x axis and still pass through (c, 0).

Fraction Challenge

$$\frac{44}{37} = 1 + \cfrac{1}{a + \cfrac{1}{b + \cfrac{1}{c}}}$$

Find the values for a, b and c given each is a positive integer.

Fraction Challenge

SOLUTION

Fraction Challenge

$$\frac{44}{37} = 1 + \frac{bc+1}{a(bc+1)+c}$$

$$\frac{7}{37} = \frac{bc+1}{a(bc+1)+c}$$

$\boxed{bc+1=7}$ $\quad\quad 7a+c=37$

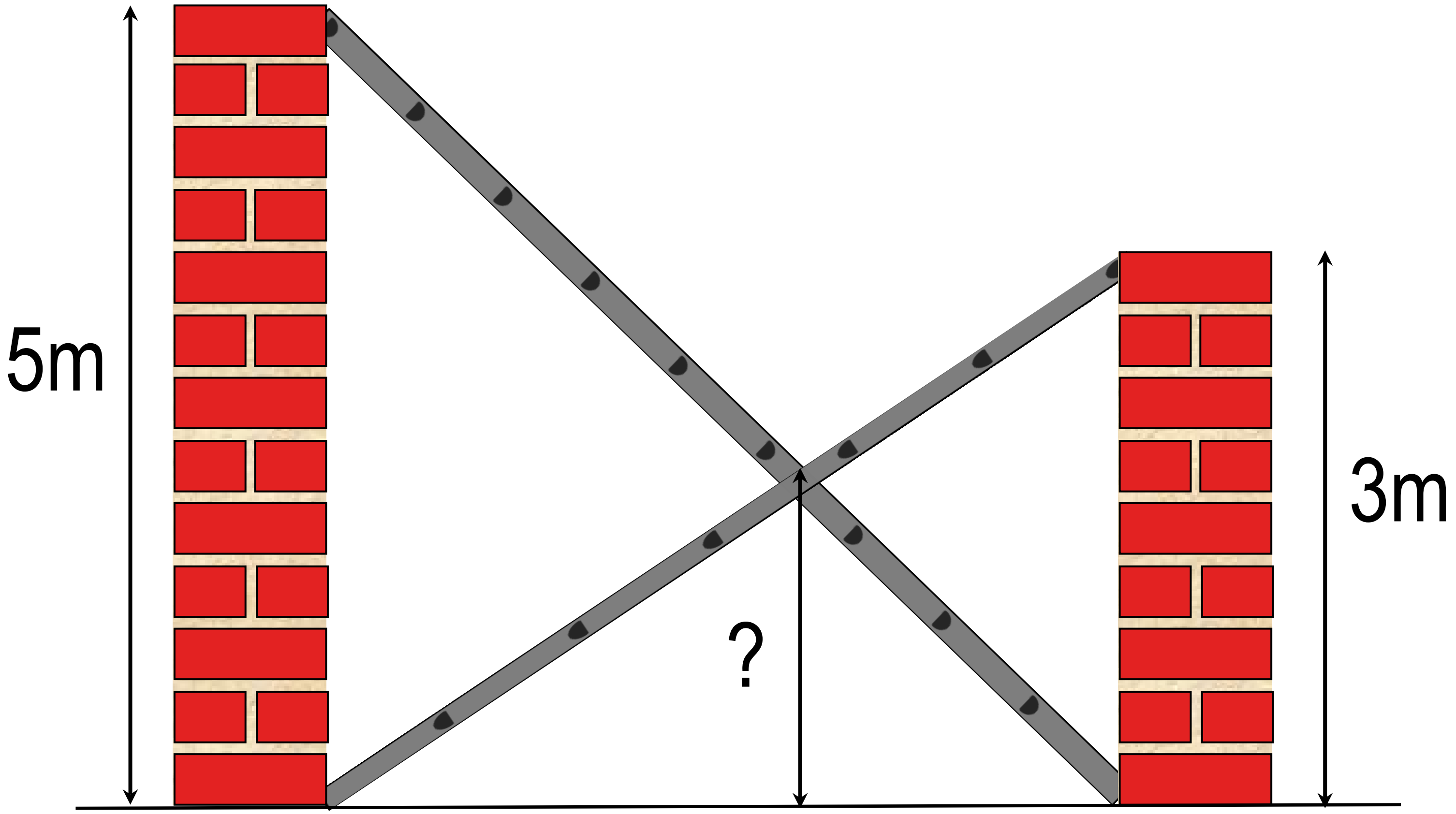
$\boxed{bc = 6 \ \therefore \text{ either } \quad b=2 \ \& \ c=3 \ \text{ OR } \ b=3 \ \& \ c=2}$

Given the options above and $7a+c=37 \Rightarrow a=5 \ \& \ c=2 \ \therefore \ b=3$

The solution can also be found by expressing $\frac{44}{37}$ as a continued fraction or visually using reducing squares.

Ladders

Two ladders lean against two walls. One wall is 5m tall, the other 3m. How high off the ground is it where the ladders cross?

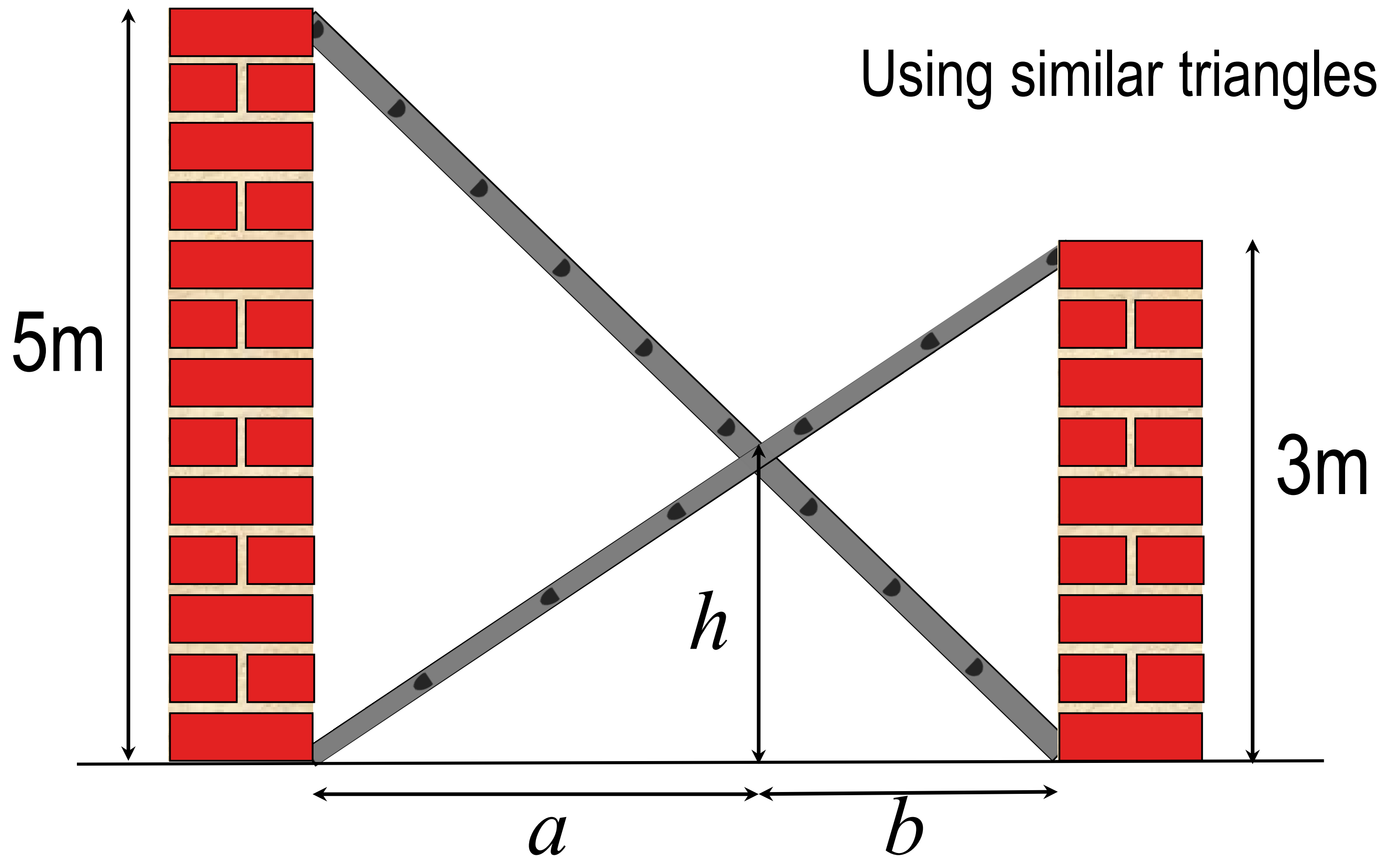


Ladders

SOLUTION

Ladders

How high off the ground is it where the ladders cross?



Using similar triangles:

$$\frac{5}{a+b}=\frac{h}{b} \quad \& \quad \frac{3}{a+b}=\frac{h}{a}$$

$$\frac{1}{a+b}=\frac{h}{5b} \qquad \frac{1}{a+b}=\frac{h}{3a}$$

$$\therefore 5b=3a$$

$$\therefore \frac{15}{8b}=\frac{h}{b} \qquad \therefore h=\frac{15}{8}$$

h is independent of a and b.

What's my Number

I have a three-digit number.

- If you add 1 to the number, the result is divisible by 3.
- If you add 2 to the number, the result is divisible by 5.
- If you add 3 to the number, the result is divisible by 7.

What is my number?

What's my Number

SOLUTION

What’s my Number

Students can start by solving an ‘easier’ problem. (Single digit)

There are no single digit numbers that satisfy the three conditions.

Clue two tells us that the number must end in a 3 or an 8.

The only two digit number to satisfy all three conditions is 53.

The next solution will be 105 + 53 = 158. **Note**: 105 = 3 x 5 x 7

Other three digit numbers will therefore be:

263, 368, 473, 578, 683, 788, 893 & 998

As easy as One – Two - Three

Nine tiles, numbered 1, 2, … 9 can be used to form three numbers, each number consisting of three digits.

- The second number is twice the first
- The third number is three times the first.

What are the three numbers?

How many solutions can you find?

1	2	3	4	5	6	7	8	9

As easy as One – Two - Three

SOLUTION

As easy as One – Two - Three

By doing some quick checks of doubling and tripling:

Let S = Smallest number: 139 < S < 329 [Can't end in 5]

Let M = Middle number: 268 < M < 658 [Number is also even]

Let L = Largest number: 378 < L < 987 [Sum of digits multiple of 3]

Examples: 123 x 2 = 246 …132 x 2 = 264, 134 x 3 =402 (2 is repeated)

Solutions:

192, 384 & 576; 219, 438 & 657; 273, 546 & 819; 327, 654 & 981

Nine Times Reverse

A four digit number is multiplied by 9, the result (answer) contains the same four digits but in the reverse order.

What is the original number?

Nine Times Reverse

SOLUTION

Nine Times Reve

$33^2 = 1089$

We can work out the 1000's digit of the origina
number and similarly with the answer.

This tells us the unit's digit of both numbers.

The 100's digit of the original number cannot cause a 'carry' digit to overflow, so it must be 0 or 1.

The multiplication of the units valu in th origi al
number (9 x 9) results in a

$99^2 = 9801$

	$\boxed{}$	$\boxed{0}$	$\boxed{8}$	$\boxed{9}$
x				$\boxed{9}$
	$\boxed{}$	$\boxed{8}$	$\boxed{0}$	$\boxed{1}$

Four Times Reverse

A four digit number is multiplied by 4, the result (answer) contains the same four digits but in the reverse order.

What is the original number?

Four Times Reverse

Solution

Four Times Reverse

The 1000's digit of the original number must be a 1 or 2 to avoid an overflow in the answer, but the units digit in the answer must be even …

2 1 7 8

The 1000's digit in the answer must be an 8 or a 9, but the units digit in the answer is a 2, so ….

x 4

The 100's digit in the original number must be a 0, 1 or 2, to avoid a carry digit, but the corresponding digit in the answer must be odd due to the carry from 4 x 8 = 32.

8 7 1 2

Four Times Reverse

Let the four digit number be: $1000a + 100b + 10c + d$

$\therefore\ 4 \times (1000a + 100b + 10c + d) = 1000d + 100c + 10b + a$

$4 \times 1000a < 10{,}000$ therefore: $a = 1\ \ or\ \ a = 2$

Also: $mod(4d, 10) = a$ is even, $\therefore\ \ a = 2$

$\therefore\ d = 8$ since $4 \times 8 = 32$

$\therefore\ 4b < 10$ so $b = 0,\ \ b = 1\ or\ b = 2$

Options: 20_8 x 4 = 8_02, 21_8 x 4 = 8_12 OR 22_8 x 4 = 8_22

21_8 is the only option … 2178 x 4 = 8712 (unique solution).

Hooray for 2024 array

If the array shown below is continued, what numbers will appear directly above and below 2024?

1

2 3 4

5 6 7 8 9

10 11 12 13 14 15 16

Hooray for 2024 array

Solution

Hooray for 2024 array

Above and below 2024?

		$\boxed{1}$		
	2	$\boxed{3}$	4	
5	6	$\boxed{7}$	8	9

...... 1936

...... 2024 2025

...... 2114 2115 2116

Each column forms a recursive sequence, also, the trailing diagonal consists of perfect squares.

Using perfect squares:

1936 is above 2024

2114 is below 2024

Down the Mountain

Down the Mountain is an 11 player game.

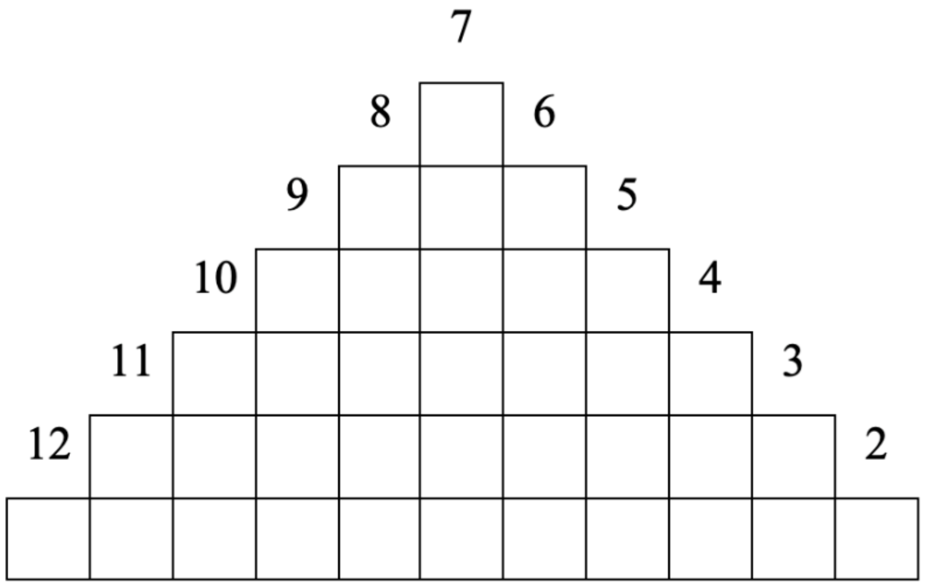
Players take it in turn to choose a number which is not already taken.

Once each person has a number, the croupier rolls two dice and adds the uppermost faces together.

A cross is then put under that number.

If a column is full of crosses that player wins, otherwise the dice are rolled again.

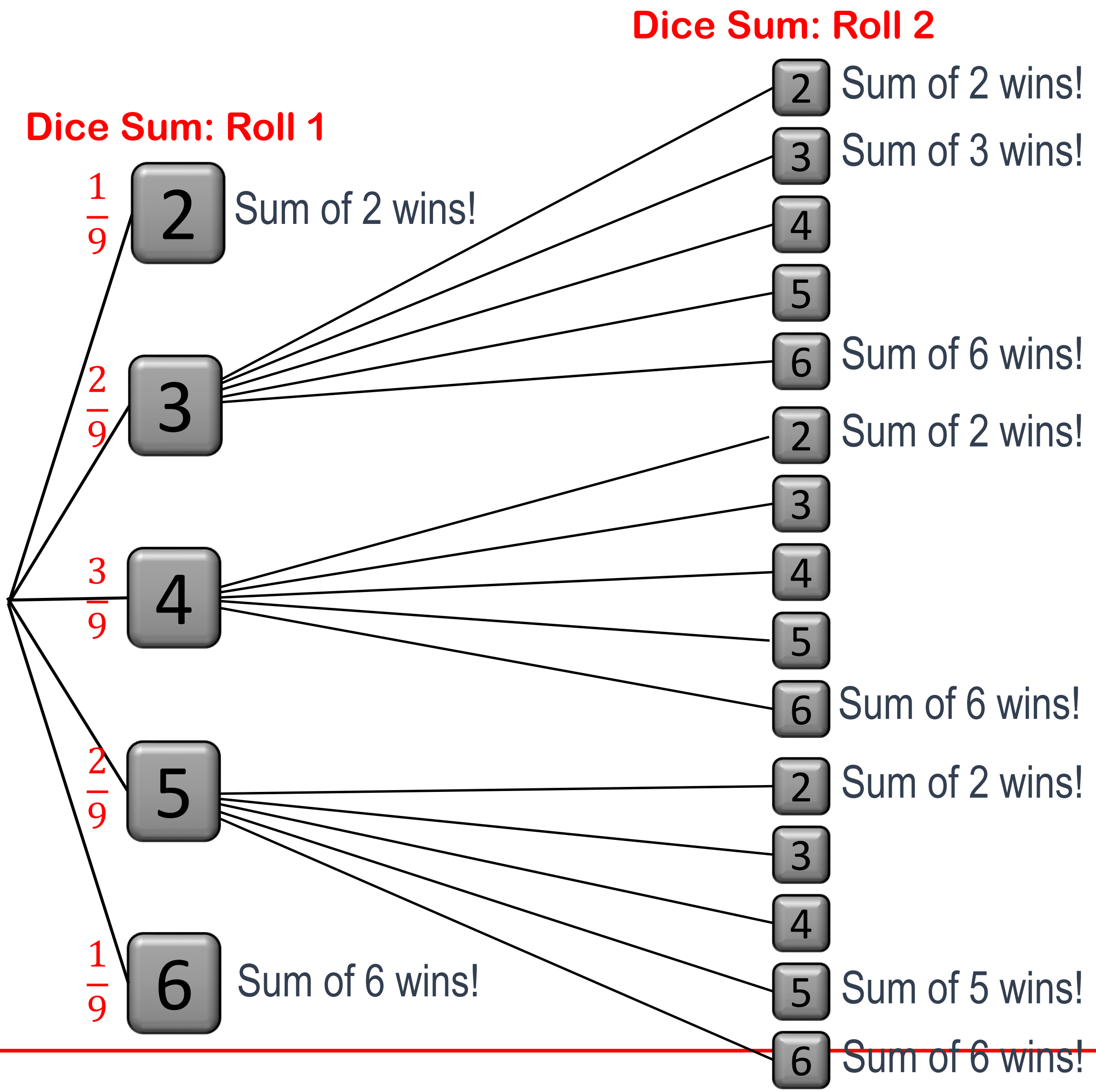
If you're the first player to choose a number, what number should you choose?



Down the Mountain

Solution

Down the Mountain



A simulation reveals all is not as it seems. A tree diagram is a great place to start, however it will be large.

Strategy: "Solve an easier problem".

A '3 sided' dice creates a significant tree, but concludes after, at most 5 rolls.

Probability of winning with a sum of 2 after n rolls:

n	1	2	3	4	5
Prob	$\frac{1}{9}$	$\frac{7}{81}$	$\frac{41}{729}$	$\frac{180}{6561}$	$\frac{432}{59049}$

Total $\approx$ 0.2885

Let X = sum of the dice:

x	2	3	4	5	6
Prob	$\frac{631}{2187}$	$\frac{344}{2187}$	$\frac{237}{2187}$	$\frac{344}{2187}$	$\frac{631}{2187}$

$\therefore$ Choose end values

Swimming for Gold

In a 200m swimming race, the swimmer in lane 5 wins, beating the swimmer in lane 6 by 20m.

The swimmer in lane 6 beats the swimmer across in lane 4 by 15m.

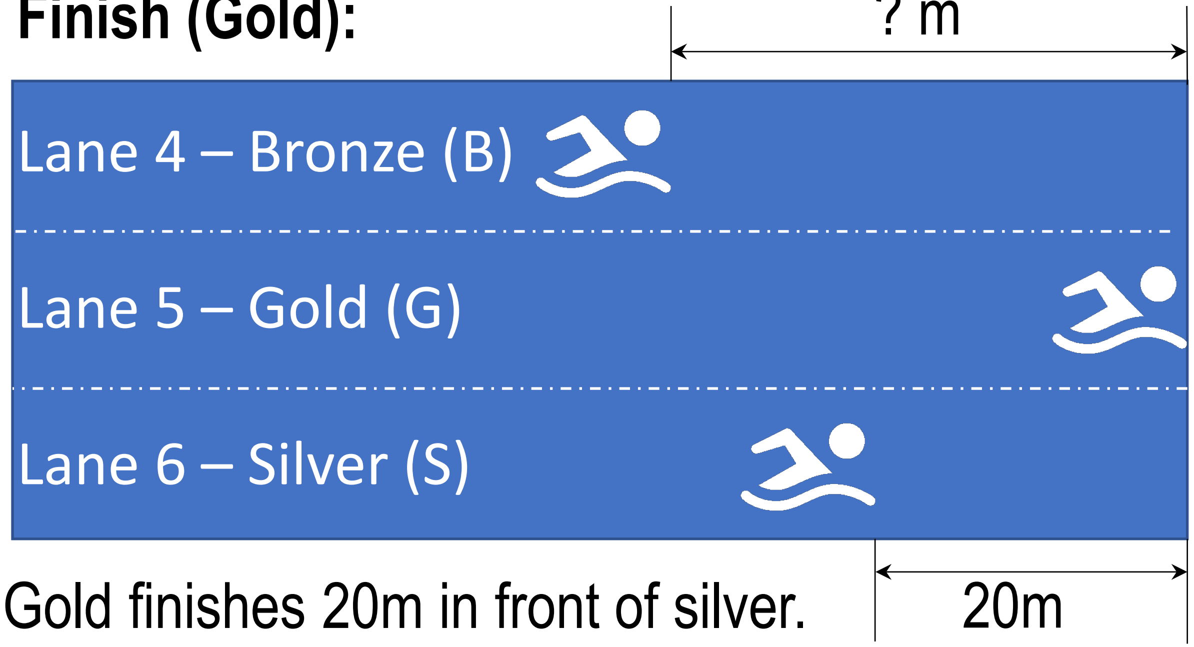
Assuming all the swimmers are swimming at a constant rate, by how many metres did the winner (lane 5) beat the swimmer in lane 4?

Swimming for Gold

Solution

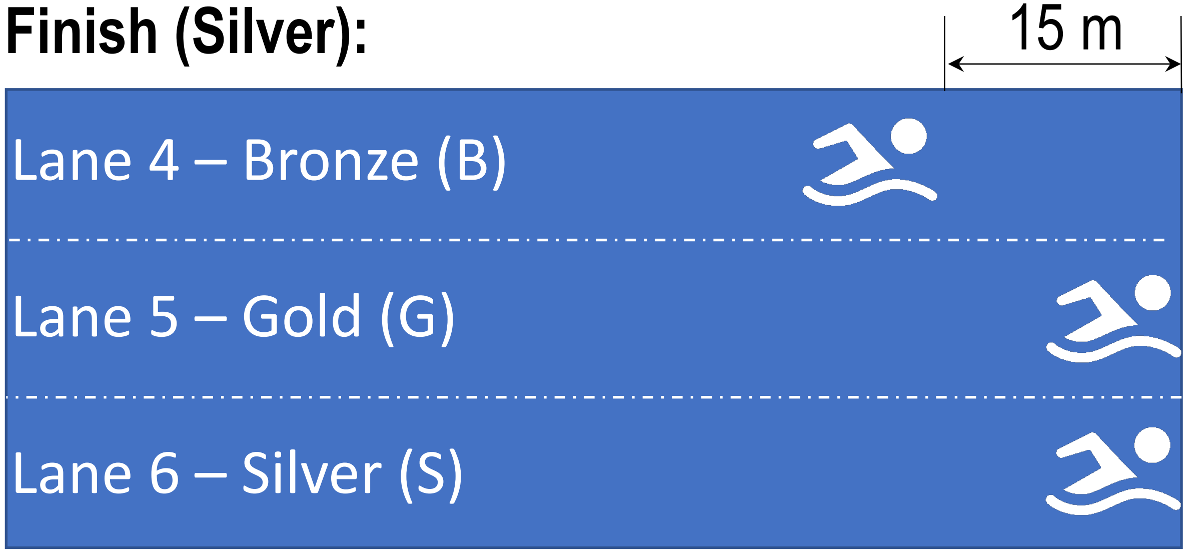
Swimming for Gold

Finish (Gold):



Gold finishes 20m in front of silver.

Finish (Silver):



Silver finishes 15m in front of bronze.

Let the winning time = 100s (it can be anything). Speed for S: $Speed = \frac{180}{100} = 1.8$m/s

Finish time for silver: $time = \frac{distance}{speed} = \frac{200}{1.8} \approx 111.11$s. Speed for B: $Speed = \frac{185}{111.11} \approx 1.666...$m/s

Position of B after 100s (when A finishes) ≈ 166.66m.